**Homework 2** Due 18:00, September 23, 2021

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# Problem 2.1

Prove the following statement by proving its contrapositive: if *r* is irrational, then *r*1*/*5 is irrational. Be sure to state the contrapositive explicitly.

**Proof: We prove the contrapositive: if *r*1*/*5 is rational, then r is rational**

Assume that *r*1*/*5 is rational, then there exists 𝑎, 𝑏∈ℤ such that:

*r*1*/*5 = where a, b are co-prime and 𝑏≠0 as it is the following, 𝑟= so if 𝑎,𝑏∈ℤ, then ∈ℤ as well. Therefore 𝑟∈ℚ

**Problem 2.2**

Prove by contradiction that log4 6 is irrational.

**Proof: We prove the contrapositive: log4 6 is rational**

where a, b are co-prime, and

6 is having 2 and 3 as devisors, 4 is having 2 as divisor, so the common divisor is 2.

But since they have a common divisor, we can write it like so:

Since the product of 2 even numbers must be even, and the product of 2 odd numbers must be odd, therefor are not equivalent, so neither b and a are, and must not be rational.

Therefore **log4 6** is irrational

# Problem 2.3

Identify exactly where the bugs are in the following bogus proof:

**Bogus Claim**: If *a* and *b* are two equal real numbers, then *a* = 0. **Proof:**

*a* = *b*

*a*2 = *ab*

*a*2− *b*2 = *ab* − *b*2

(*a* − *b*)(*a* + *b*) = (*a* − *b*)*b // a-b=0 -> division by 0*

*a* + *b* = *b is undefined*

*a* = 0*.*

**Problem 2.4**



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Write out ((*α* ∨ *β*) ∧ (*β* → *α*) using only ¬ and → *.*

# Problem 2.5

Write the truth table for the following formulas (keep in mind the precedence rules):

1. ((*A*1∧ *A*2) → *A*3) → (*A*1∨ *A*2);

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | *∨* | ((*A*1∧ *A*2) → *A*3) → (*A*1∨ *A*2) |
| T | T | T | T | T | T | T |
| T | T | F | T | F | T | T |
| T | F | T | F | T | T | T |
| F | T | T | F | T | T | T |
| T | F | F | F | T | T | T |
| F | T | F | F | T | T | T |
| F | F | T | F | T | F | F |
| F | F | F | F | T | F | F |

1. (¬*A*5 → *A*2) → (*A*5∧ *A*2);

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  | *A5*∧ *A*2 |  |
| T | T | F | T | T | T |
| F | T | T | T | F | F |
| T | F | F | T | F | F |
| F | F | T | F | F | F |

1. (*A*1∧ *A*2 → *A*3∧ *A*4) ←→ (¬(*A*1∨ *A*3) →¬(*A*2∨ *A*4)).

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|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |
| T | T | T | T | T | T | T | F | F | F | T |
| T | T | T | F | T | F | F | F | F | F | F |
| T | T | F | T | T | F | F | F | F | F | F |
| T | F | T | T | F | T | T | F | F | F | T |
| F | T | T | T | F | T | T | F | F | F | T |
| T | T | F | F | T | F | F | F | F | F | F |
| T | F | F | T | F | F | T | F | F | F | T |
| F | F | T | T | F | T | T | F | F | F | T |
| T | F | T | F | F | F | T | F | T | T | T |
| F | T | F | T | F | F | T | T | F | F | F |
| F | T | T | F | F | F | T | F | F | T | T |
| F | F | F | T | F | F | T | T | F | F | F |
| F | F | T | F | F | F | T | F | T | T | T |
| F | T | F | F | F | F | T | T | F | F | F |
| T | F | F | F | F | F | T | F | T | T | T |
| F | F | F | F | F | F | T | T | T | T | T |

**Problem 2.6**

Show that (*α* → *β*) ∨ (*β* → *α*) is a tautology.

A formula φ is called a tautology, if it is satisfied by every truth assignment.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  | (*α* → *β*) ∨ (*β* → *α*) |
| T | T | T | T | T |
| T | F | F | T | T |
| F | T | T | F | T |
| F | F | T | T | T |

**Problem 2.7**

Prove Contrapositive and Composed Contrapositive Inference Rules.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
| T | T | F | F | T | T |
| T | F | F | T | F | F |
| F | T | T | F | T | T |
| F | F | T | T | T | T |

The proof consist in the equivalence of the truth table

# Problem 2.8

Is the following logical deduction true?

If detective Jones haven’t seen Smith last night, then either Smith is the murderer, or Jones is lying. If Smith is not the killer, then Jones haven’t seen Smith last night and the murder happened after midnight. If the murder happened after midnight, then either Smith is the murderer or Jones is lying. Therefore, Smith is the murderer.

*α = {*Jones haven’t seen Smith last night}

*β = {*Smith is the murderer} -> = {Smith is not the killer}

= {Jones is lying}

= {the murder happened after midnight}

=>

**=**1(true)

or , while both cannot be simultaneous

(explanation to the previous assumption - if or =>

So as a result we have --- , therefor ->

If the murder happened after midnight, then either Smith is the murderer or Jones is lying.

Mean that logical deduction is true,

as Jones is lying,,as as*{*Jones haven’t seen Smith last night}=  *,* Therefore, Smith is the murderer.

# Problem 2.9

Answer the following questions: =>

1. Write the negation of the statement: “All prime numbers are odd.”

“Not all prime numbers are odd.”

1. Write the contrapositive of the statement: “If a set is finite, then it is countable.”

“If a set isn’t countable, then it is infinite.”

1. Write the converse of the statement: “You pass the exam, if you study hard.”

“If you study hard, you pass the exam.”

1. Write down the negation of the statement: “There are people who do not study Special Math.”

“There are people who study Special Math.”

1. Write down the negation of the statement: ∀x ∈ R ∃y ∈ Z such that (x + y) ≥ 0. Is the original statement true or false?

” Yes, because Z∈R, so ∀x ∈ R ∃ y≥0∩y≥|x|”